$$K_{\alpha\beta\gamma}(t)=\beta\Gamma(\alpha)^{-1}L_G(t)\int_a^{k(t)}\frac{L_f(y)L_m(y)}{y}\,\mathrm{d}y \quad \text{if } \beta+\gamma=0.$$

The result for the case $\alpha+\beta+\gamma=0$ is also obtained.

On Asymptotic Behavior of Transition Density of Markovian Diffusion Processes with Small Diffusion: A Stochastic Control Approach

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Let $x(\cdot)\equiv x^\varepsilon(\cdot)$ be the diffusion given by

$$\mathrm{d}x(t)=b(x(t))\,\mathrm{d}t+\varepsilon\sigma(x(t))\,\mathrm{d}w(t),$$
$$x(0)=x\in R^n.$$

We are interested in the asymptotic behavior of the transition density $P_t^\varepsilon(x,y)$ of $x^\varepsilon(\cdot)$. A stochastic control method is proposed to study $P_t^\varepsilon(x,y)$ as $\varepsilon\to 0$. As a consequence of this, we will obtain a Ventsel-Friedlin type result:

$$\lim_{\varepsilon\to 0}\varepsilon^2\log P_T^\varepsilon(x,y)=-\inf_{\substack{\phi(0)=x\\ \phi(T)=y}}\tfrac{1}{2}\int_0^T\|\dot\phi(t)-b(\phi(t))\|^2\,\mathrm{d}t$$

where $\|\dot\phi-b(\phi)\|^2=\sum a^{ij}(\phi)(\dot\phi_i-b_i(\phi))(\dot\phi_j-b_j(\phi))$ and $(a^{ij})=(\sigma\sigma^*)^{-1}$.

The Double Points of a Diffusion

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Consider a Markov process in R^d, with continuous paths and specified transition density functions. Under a set of hypotheses on the latter, we prove that almost all sample paths have double points. Then, we show that the diffusion in R^2 or R^3 generated by a non-degenerate elliptic operator has double points a.s.; this extends the classical results of Dvoretzky et al. for Brownian motions.

Random n-Simplices and a Central Limit Theorem

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Let $B_1,\ldots,B_n$ be independent copies of the Brownian motion on the d-dimensional torus. By virtue of the flatness of the torus we can define the Brownian random n-simplex $\boldsymbol{B}(V)$ for each n-cube V by

$$\boldsymbol{B}(\boldsymbol{t})=B_1(t_1)+\cdots+B_n(t_n),\quad \boldsymbol{t}=(t_1,\ldots,t_n)\in V.$$

Take a smooth n-form a on the torus and consider the stochastic integral

$$\int_{B(V)} a = \int \cdots \int_V ({}^\circ\mathrm{d}B_1^{i_1})_{t_1} \cdots ({}^\circ\mathrm{d}B_n^{i_n})_{t_n} a_{i_n \cdots i_n}(\boldsymbol{B}(t))$$

in the Stratonovich sense. One can also consider the stochastic integral of the form a in the Ito sense which will be denoted by Mart-$\int_V a$.

Lemma. *Let $a^e = \mathrm{d}b$ be the exact part of the n-form a in the de Rham-Kodaira decomposition. Then,*

$$\int_{B(V)} a = \text{Mart-}\int_{B(V)} (a - a^e) + \text{Mart-}\int_{B(\partial V)} b.$$

Theorem. *The law of $|V|^{-1/2} \int_{B(V)} a$ converges to the normal law $N(0, \|a - a^e\|^2)$ as V tends to R^n in the sense of van Hove, while the law of $|V|^{-1/2}$ Mart-$\int_{B(V)} a$ converges to the law $N(0, \|a\|^2)$.*

The convergence can be proven in the sense of current valued random processes. This result suggests the possibility of the generalization of the results on 1-forms (to n-forms on arbitrary compact Riemannian manifolds) which is obtained in: Y. Ochi, Limit theorems for a class of diffusion processes. Cf. also, N. Ikeda, Central limit theorems and random currents.

On the Existence of Intersectional Local Time Except on Zero Capacity Set

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Let (W, μ) be the d-dimensional standard Wiener space, $d \geq 2$. Consider a capacity on W, induced by the W-valued Ornstein-Uhlenbeck process X_τ. A subset of W has zero capacity if it is a polar set with respect to X_τ. We prove that a functional $\psi(\alpha, w)$ on W defined by

$$\psi(\alpha, w) = [(d-\alpha)/4] \int_0^1 \int_0^1 |w(s) - w(t)|^{-\alpha} \,\mathrm{d}s\,\mathrm{d}t \quad (\alpha < 2)$$

is finite not only μ-almost surely but also quasi-everywhere, i.e. except on a set of zero capacity. This means that the Hausdorff dimension of range of w is no less than 2 quasi-everywhere.

Furthermore, $\Psi_n(w)$ defined by $\Psi_n(w) = \psi(2 - 2^{-n}, w) - 2^n$ is shown to converge not only μ-almost surely but also quasi-everywhere. In case $d = 2$, the limit functional is symbolically expressed as

$$(\pi/2) \int_0^1 \int_0^1 \delta(w(s) - w(t)) \,\mathrm{d}s\,\mathrm{d}t - C,$$

where C is an infinite constant and δ is the Dirac δ-function.